Name: ________________
Student Number: ________________

EXAM on WBPH030 “Solid State Physics”

Content: 5 pages (including this cover page)

Thursday 26 January 2023; Exam Hall

Exam drafted by (name first examiner): Jianting Ye

Exam reviewed by (name second examiner): Graeme Blake

For administrative purposes; do NOT fill the table

	Maximum points	Points scored
Question 1	26	
Question 2	26	
Question 3	22	
Question 4	26	
Total	100	

Final mark: _________

1. Crystal Structure and X-ray scattering

1.1. (2p) How to form a crystal using the concepts of lattice and a basis?

1.2. (3p) There are 3 space lattices of cubic type. Name and sketch them (rough sketches without using rulers are acceptable).

1.3. (6p) Define the NaCl lattice type and coordinates of its basis in terms of $\boldsymbol{r_j} = x_j\boldsymbol{a_1} + y_j\boldsymbol{a_2} + z_j\boldsymbol{a_3}$, where the $\boldsymbol{a}_i$ are vectors in the Cartesian coordinate system. What is the maximal packing fraction for this lattice type (considering all the atoms to be of the same radius)? What are Miller's indices of the most densely packed plane?

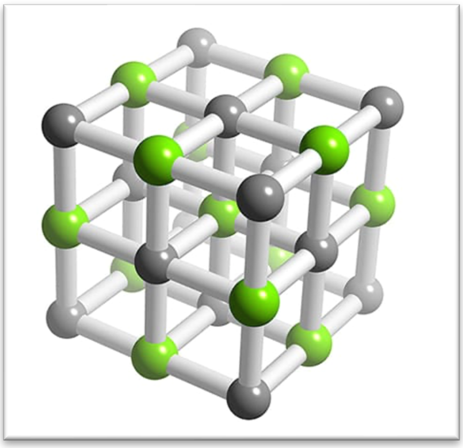

Figure 1. NaCl crystal structure

1.4. (5p) Briefly discuss the Bragg diffraction. What is the requirement for the wavelength of the radiation to be diffracted on a crystal?

1.5. (10p) Determine the structural factor of the FCC lattice of KCl.
If the atomic form factors of both K and Cl are the same, one will obtain the *x*-ray diffraction pattern similar to simple cubic. Explain why.
The structural factor is given by:

$$S_G = \sum_j e^{-i\boldsymbol{G}\boldsymbol{r}_j} \int dV\, n_j(\boldsymbol{\rho})\, e^{-i\boldsymbol{G}\boldsymbol{\rho}}.$$

2. Phonons and Electrons

2.1. (4p) Write down the Newtonian equation of a 1D chain made of monoatomic harmonic oscillators.

2.2. (4p) Discuss what will happen if there are two types of atoms. Sketch the dispersion relation for phonons in this chain. Discuss the interaction of such a lattice with visible light.

2.3. Consider **a 3D simple cubic** lattice of a metal.

2.3.1. (9p) Determine the phononic heat capacity, at low temperature, in Debye approximation if the dispersion relation is assumed to be: $\omega = v_s|\boldsymbol{k}|$.

2.3.2. (9p) Derive the density of electronic orbitals in this lattice if the dispersion relation for the free electron is given as:

$$E = \frac{\hbar^2}{2\mathrm{m}} k^2.$$

Derive the expression for the radius of a Fermi surface if the density of electron is n.

3. Nearly free electrons, intrinsic and impurity conductivity of semiconductors

3.1. (4p) Discuss the reason for forming energy gaps at the Brillouin zone (BZ) boundary. Sketch and explain the difference between the metal, semiconductor, and insulator.

3.2. (6p) Compare the bands folded to the first BZ by $\boldsymbol{k}_{1BZ} = \boldsymbol{k} \pm n\boldsymbol{G}$, where n is an integer

Which band (n = 0, 1, or 2 …) could have the smallest effective mass?

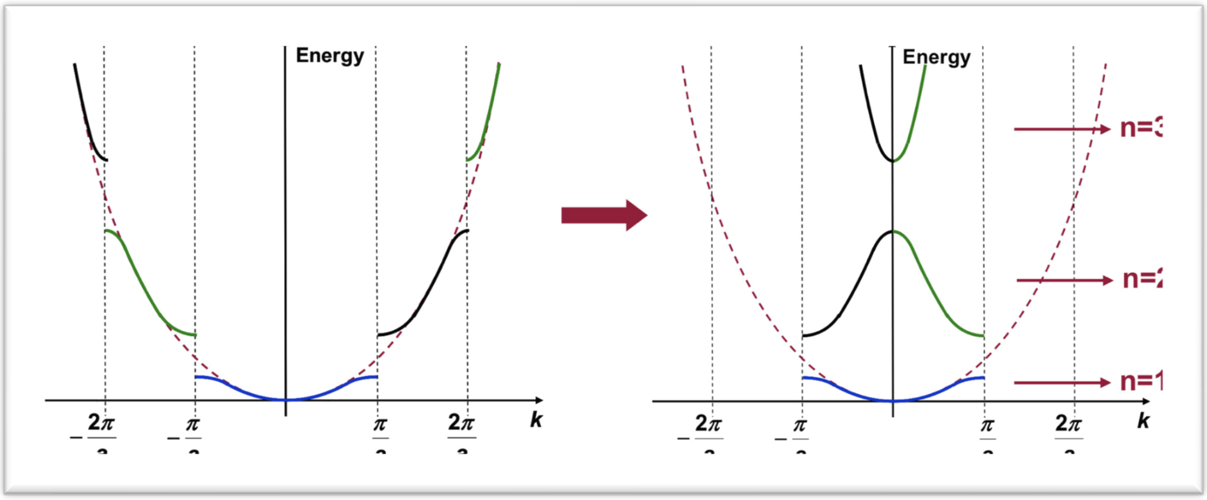

Figure 2 Schematics of the zone folding

3.3. (6p) How to excite carriers in an intrinsic semiconductor?

3.4. (6p) Contacting semiconductor, say Si with n- and p-type impurities, one can form a ***p-n*** junction. Show the the processes of band realignment at thermal equilibrium. How will the energy bands can conductivity will change at positive and negative biases (positive bias V is applied from the p- to the n- size).

4. Crystals in a magnetic field

4.1. (6p) Using Drude's model, briefly explain the Hall effect in metals.

4.2. (4p) Which physical parameter defines para- or diamagnetic? How do those two types of materials behave in a magnetic field?

4.3. (6p) Some materials demonstrate a superconducting transition at a critical temperature. Describe the difference in magnetization between a superconductor and a perfect conductor under an external magnetic field.

4.4. (10p) For the system shown on right, write down the expression for the magnetic moment of a system subjected to a magnetic field ***B***.

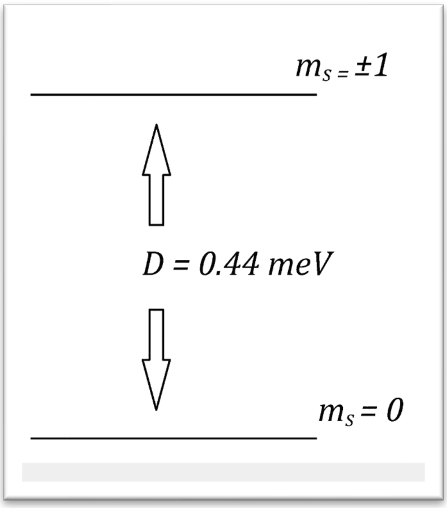

Figure 3. Triplet state system, for example, the ground state of O_2.

--------------------------------- End of Questions ---------------------------------

FORMULA LIST

The density of states is defined by:

$$D(\omega) = \frac{dN}{d\omega}$$

Fermi-Dirac distribution:

$$f(\varepsilon)_{FD} = \frac{1}{e^{\frac{\varepsilon-\mu}{k_B T}} + 1}.$$

Bose-Einstein distribution:

$$f(\varepsilon)_{BE} = \frac{1}{e^{\frac{\varepsilon-\mu}{k_B T}} - 1}.$$

Quantum statistical average:

$$< A > = \frac{\sum_i A_i e^{-\frac{E_i}{k_B T}}}{\sum_i e^{-\frac{E_i}{k_B T}}}.$$